Can non-extremal Reissner-Nordström black hole become extremal by assimulating infalling charged particle and shell ?

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Abstract

By using the gedanken experiments suggested by Bekenstein and Rosenzweig, we have shown that non-extremal Reissner-Nordström black hole cannot turn into extremal one by assimulating infalling charged particle and charged spherical shell

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It was traditionally believed that the extremal black hole is the limiting case of its nonextremal counterpart, when the inner Cauchy horizon r_- and outer event horizon r_+ degenerate, the nonextremal black hole becomes extremal [1,2]. At the extremal limit, it has been pointed out by many authors [3-8] that a phase transition and the corresponding scaling law exists.

But this traditional viewpoint has been challenged by many workers [9-14] recently. Based on an argument of the topological difference between the extremal and the nonextremal Reissner-Nordstrӧm (RN) black hole, Hawking et al. [9] shew that the entropy of extremal RN black hole is zero and the formula of Bekenstein-Hawking entropy $S = A/4$, where A is the surface area of the horizon, is not valid. Ensuring the stability against Hawking radiation, the temperature of the extremal black hole (EBH) is zero and be different from the case of nonextremal black hole (NEBH). They claimed that the entropy changes discontinuously in the extremal limit implies that one should regard NEBH and EBH as qualitatively different objects and a NEBH cannot turn into EBH. Furthermore, after calculating the timelike distance between r_+ and r_-, 't Hooft [15] argued that there is no extremal limit. He said that the macroscope EBHs are physically illdefined limits, it would be a mistake to treat an extremal black hole horizon as just one horizon. On the other hand, especially in the string model [13] or employing the brick wall model to calculate the entropy of scalar field [11], many other authors [11-14] have shown that the extremal limit still exist and the Bekenstein-Hawking entropy formula is still valid. However, recently the details of what makes an extremal black hole different from a nonextremal black hole has also been observed as of particular importance in string theory [16]. All these investigations indicate that the entropy and other physical properties are still far from fully understood in the extremal limit.

To clarify this puzzle, it is of important to study whether one can use some physical processes such as radiation, absorption etc. to realize the extremal limit and make the NEBH becomes EBH. The horizons of RN black hole locate at

$$r_\pm = M \pm \sqrt{M^2 - Q^2} \tag{1}$$

where M and Q are the mass and charge of the hole respectively. For NEBH, $M > Q$; for EBH, $M = Q$. Obviously, if a physical process can increase the charge Q or decrease the mass M of the hole, one can realize the extremal limit and transform the NEBH into

EBH. It was shown in [17] that the radiational process such as Hawking radiation, Penrose process and super-radiance cannot transform the NEBH into an extremal one. In addition to radiation, the black hole has another important character, namely, absorption. Can the tremendous gravitation of the RN black hole be used to swallow charges and make the NEBH becomes extremal? This is the objective that we hope to discuss in this paper.

In the last few years, an appealing problem whether one can push the black hole over the brink to violate the extremal condition and remove the event horizon was addressed by many authors. The first negative answer to this problem was put forward by Hiscock [18]. A quite similar approach of studying the problem but with some quantum point of view was suggested by Bekenstein and Rosenzweig (BR)[19], in which two gedanken experiments to investigate the stability of the event horizon were proposed. In their reference, an attempt was made to violate the condition $Q^2 \leq M^2$ by adding to an extremal RN black hole entity which has charge q in such a way that $q^2+Q^2 > (M+mE)^2$, where $Q \in U(1)$ and $q \in U(1)'$, $U(1)$ and $U(1)'$ are different gauge fields. E is specific energy and m the mass of the infalling entity. The entity can be a spherical shell or a point charge. They addressed different kinds of processes carrying both classical and quantum machanical entities and studied whether this charge can enter the RN black hole and break the extremal condition. They found that all processes in which such entities can be used to remove the event horizon are forbidden. The same result was also obtained by Jensen [20] with an attempt by adding matter with a negative gravitational energy to reduce the mass term of RN black hole. In a previous paper [21], we extended the above studies to three dimentional Banados, Teitelboim and Zanelli black holes. In this paper, we will employ the BR gedanken experiments to study another problem, namely, whether the extremal condition can be realized. We will prove that not only processes which break the extremal condition are forbidden, but also processes which make

$$q^2 + Q^2 = (M + mE)^2 \tag{2}$$

and transform the NEBH into EBH are forbidden as well. A nonextremal RN black hole cannot assimulate the infalling charged particle and/or charged shell to become an EBH from these gedanken experiments.

1.Infall of charged shell

Following [19], the exterior metric for a spherical distribution with two different $U(1)$

charges Q and q is

$$ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2+q^2}{r^2})dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \frac{Q^2+q^2}{r^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

The equation of motion of a particle which locates at the outer edge of the shell is

$$(\frac{dr}{d\tau})^2 - \frac{2}{r}[M + mE(1 - \frac{q^2}{m^2})] + \frac{1}{r^2}[Q^2 + q^2(1 - \frac{q^2}{m^2})] = E^2 - 1 > 0 \tag{4}$$

Where E is fixed to $E > 1$ for all particles [19]. Defining the terms following $(\frac{dr}{d\tau})^2$ as the effective potential, we see there is a potential barrier outside the horizon and the maximum value

$$V_{max} = \frac{[mE(q^2/m^2 - 1) - M]^2}{q^2(q^2/m^2 - 1) - Q^2} \tag{5}$$

is located at

$$r_e = \frac{q^2}{mE} + \frac{Mq^2/mE - Q^2}{mE(q^2/m^2 - 1) - M} \tag{6}$$

Since we begin with a nonextremal RN black hole, satisfying

$$M^2 > Q^2 \tag{7}$$

and hope the resulting configuration becomes extremal and satisfy Eq(2) after the assimulation of the charged shell. Combining Eq(2) and Eq(7), we can write

$$q^2 = m^2E^2 + 2MmE + m^2\alpha,\ Q^2 = (1 - \beta^2)M^2 \tag{8}$$

where $\alpha > 0$ and $0 < \beta < 1$. Thus we obtain

$$r_e > 2M \tag{9}$$

and

$$V_{max} - (E^2 - 1) = m^2 \frac{(E^2\beta - E^2 - \beta)M^2 - (E^2m^2 - m^2 + \alpha m^2 + 2EmM)\alpha}{Q^2m^2 - q^4 + m^2q^2} > 0 \tag{10}$$

Eq(9) means the shell will reach the potential barrier before reaching the horizon. Eq(10) means the shell must reach a turning point before reaching the maximum of the potential. The potential will prevent the shell from falling into the black hole to transform a NEBH to EBH.

2.Infall of point charge

Suppose the point charge with mass m and charge q is a test particle, its equation of motion is

$$(\frac{\mathrm{d}r}{\mathrm{d}\tau})^2 - \frac{2M}{r} + \frac{Q^2}{r^2} = E^2 - 1 > 0 \tag{11}$$

because of $m \ll M, q \ll Q$. We can easily prove that in this case the potential cannot prevent the particle from falling into the black hole.

But we notice another mechanism. If the resulting configuration becomes extremal after absorbing the test particle, the charge q must satisfy

$$q^2 = M^2 + 2MmE + m^2E^2 - Q^2 > 2MmE + m^2E^2 \tag{12}$$

Similar to the classical electron radius, the classical charge radius is

$$r_c = q^2/m > 2ME + mE^2 > 2M \tag{13}$$

by using Eq(12). If the point charge infalls into NEBH and turn it into EBH, its radius must be bigger than that of the original black hole radius. By using the same argument of ref.[19], we conclude that the attempt of transforming NEBH into EBH by assimulating point charge is impossible.

In summary, employing two gedanken experiments suggested by Bekenstein and Rosenzweig, we examine the possiblity whether the nonextremal RN black hole can be transformed into the extremal one by assimulating infalling charged particle and shell. All results are negative. Besides Hawking radiation, Penrose process and super-radiance, the absorption of charged particle and shell cannot transform the NEBH into EBH. Even though the Bekenstein-Rosenzweig gedanken experiments cannot rule out all absorptive processes, but as examples, it seems to support the argument of Hawking et al. [9], EBH is a different object from that of NEBH and can not be developed from the NEBH.

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